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Prepared for

National Aeronautics and Space Administration
Washington 25, D.C.

Contract No. NASw-698

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Quantum Physics Laboratory

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PHYSICAL RESEARCH DIVISION

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THEORETICAL STUDY OF THE COUPLING BETWEEN THE SOLAR WIND AND THE EXOSPHERE

I. PROGRESS OF WORK

During this period the model discussed previously was improved and detailed comparison was made between our predictions and the Explorer 12 and Pioneer 5 observations. Although several important features of the data remain to be explained, our confidence in the current instability-fast-diffusion model of the magnetopause is now increased, and a considerably expanded account of our theory is attached. It is expected that this report will be published in the Journal of Geophysical Research. A talk on this work ("Plasma Instabilities in the Magnetopause") will be delivered at the Magnetopause session of the American Geophysical Union meeting in Boulder, December 26-28, 1963.

II. KEY PERSONNEL

During this period F. Scarf and R. Fredricks worked actively on this problem, with assistance from B. Fried and W. Bernstein. L. M. Noble and A. Peskoff also participated.

III. REPORTABLE ITEMS

None.

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ATTACHMENT: Preprint - "A Model for a Broad, Disordered Transition Between the Solar Wind and the Magnetosphere" revised version, December, 1963.

PREPRINT

A MODEL FOR A BROAD DISORDERED TRANSITION
BETWEEN THE SOLAR WIND AND THE MAGNETOSPHERE
(Revised)

W. Bernstein,* R. W. Fredricks† and F. L. Scarf†
(TRW Space Technology Laboratories, Redondo Beach, Calif.)

15531 ab

It has been suggested that charge separation electric fields in the Chapman-Ferraro sheath generate currents which are large enough to trigger the two-stream plasma instability. We argue that as the electric field energy saturates, the drift energy is lowered and the electron temperature rises so that the instability is generally not quenched. Instead, it can change form leading to growing ion waves. The equilibrium state then involves fluctuating electromagnetic fields which allow "fast" plasma diffusion across the main magnetic field. In this case, the exospheric thermal plasma, together with that part of the solar wind plasma which has attained energy equipartition, can form current systems leading to a broad, disordered transition region between the magnetosphere and the solar wind.

Author

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1. INTRODUCTION

The traditional description of the solar wind-magnetosphere interface is based on the static Chapman-Ferraro model of a "thinnest" transition from a compressed geomagnetic field to a completely diamagnetic solar plasma. It is frequently tacitly or explicitly assumed that the transition region is stable and that the minimum sheath is inevitable (see Blum, 1963). Both of these statements are generally incorrect.

Even if one neglects the hydromagnetic instability discussed several years ago by Dungey (1954) and Parker (1958a) and the long range Hall electric fields emphasized mainly by Alfvén, the sheath has an intrinsic plasma instability of the two-stream variety associated with charge separation electric fields produced by the electron-proton mass difference. In this note we argue that nonlinear effects do not quench the instability; as the drift energy is lowered and the electron temperature rises, the drift threshold for ion sound wave instability drops rapidly. If the equilibrium configuration is indeed unstable with respect to ion sound wave generation, then various mechanisms which greatly broaden the sheath become operative. The fluctuating electric fields allow "fast" or "Bohm" diffusion of plasma across the magnetic field (Spitzer, 1956, 1960). The sheath of diffusely reflected particles broadens and drifts inward, and the whistler plasma within the geomagnetic cavity diffuses outward. Moreover, since the exospheric plasma and the diffusely deflected particles have "orbits" which are completely different from those assumed in the C-F model, there is no reason to believe that the final self-consistent sheath resembles the thinnest one. In fact, Grad (1961) has demonstrated that the thinnest self-consistent transition will not form even in the stable case of hot equal mass protons and electrons unless there are no "trapped" or "quasi-trapped" particles in the transition region.

In the next section the instability of the standard Chapman-Ferraro sheath is demonstrated. Section 3 contains a discussion of a possible broad self-consistent equilibrium state in which the distinction between solar wind ("free") and "trapped" plasma plays an important role. This configuration compares well with some experimental data but it is noted that certain large scale aspects of the magnetosphere-wind interaction should not be completely explainable in terms of a simple model which ignores the earth's rotation, space charge formation, wind inhomogeneities, etc. The last section contains a brief statement of our motivation in trying to construct a new model of the broad disordered interface. In particular, some of the doubtful features of the "collisionless shock" model are discussed.

2. PLASMA INSTABILITIES

In the Chapman-Ferraro model of the geomagnetic cavity (Blum, 1963), it is assumed that the incident solar wind particles move in straight lines up to some regular boundary of the earth's field where they are specularly reflected and returned to the stream in a new direction. The net pressure transmitted is $2N_o m u_o^2 \cos^2\psi$ ($m = m_p + m_e \simeq m_p$, u_o is the free solar wind speed, $|\underset{\sim}{u}_o \times \underset{\sim}{B}| = u_o B \sin\psi$, N_o is the free solar wind density, and the factor of 2 is inserted because at the boundary we have particles coming from and returning to the stream); this must equal $B^2/8\pi$, the pressure of the total (tangential) magnetic field at the boundary of the cavity. At the subsolar point, ($r = r_o, \psi = 0$) it is easy to demonstrate that $B(r_o) = 2B_G(r_o)$, where B_G is the undisturbed geomagnetic field. The field falls to zero at $r = r_o + \delta$, and the idealized diamagnetic profile modified by addition of a small interplanetary field, B_I, is shown in Fig. I.

The uniqueness of this minimum profile, its stability, and the size of the transition region must be investigated before one attempts a valid comparison between theory and experiment. To a large extent these complex points are interrelated, but they have been explored individually using simple one dimensional models. For instance, consider an unperturbed field, $\underset{\sim}{B}^o = \underset{\sim}{B}_z^o(x)\underset{\sim}{i}_z$, $B_z^o(x) \longrightarrow 0$, $x \longrightarrow + \infty$, with a fully ionized plasma of equal mass particles incident from $x = \infty$; here $(\dot{x})^2 = u_x^2(x) \longrightarrow u_o^2$, $N(x) \longrightarrow N_o$ as $x \longrightarrow + \infty$, $\dot{y} = u_y(x) \longrightarrow 0$, $x \longrightarrow + \infty$, $\dot{z} = 0$, all x. If there are no collisions and no electric fields, then each particle travels in a well defined trajectory with constant energy, $\mathcal{E} = \frac{1}{2}\, m(u_x^2(x) + u_y^2(x))$, and constant canonical momentum, $P_y = mu_y(x) \pm eA_y(x)/c$, where $dA_y(x)/dx = B_z(x)$, and B_z is the <u>total</u> (external, B_z^o, plus induced B_z') magnetic field. However, the self-consistent solution requires $dB_z'(x)/dx = 4\pi j_y(x)$ with

$$j_y(x) = \frac{2e}{c} \int dv_x dv_y v_y f(v_x, v_y, x)$$

and f is the local Boltzmann velocity distribution function for the plasma particles. Under these conditions, the self-consistent solution is unique. Furthermore, the magnetic profile can be easily evaluated since $f(v_x, v_y, x) = f^*(\mathcal{E}, P_y)$ so that j_y depends on x only through its dependence on A(x). The induced vector potential is given by $d^2A'(x)/dx^2 = 4\pi j_y\left[A'(x) + A^o(x)\right]$, and the self-consistent field is uniquely determined in terms of $A^o(x)$, the unperturbed field, and $f^*(\mathcal{E}, P_y)$, the asymptotic distribution function for the incident particles. Grad (1961) has shown that the completely diamagnetic C-F sheath is then obtained at $x = x_o$ with $Nmu_o^2 = B^2(x_o)/8\pi$, $(B(x_o) = 2B^o(x)_o$, $N(x_o) = 2N_o)$, and that the transition region, δ, is generally on the order of c/ω_p with $\omega_p^2 = 4\pi Ne^2/m$. The pressure balance relation yields the alternate form, $\delta \approx u_o/\omega_c, \omega_c = eB(x_o)/mc$.

This interaction is modified if the plasma consists of particles of unequal mass such as protons and electrons. In this case the protons tend to penetrate farther than the electrons and an electric field develops along the x-axis. This field accelerates the electrons and causes them to execute broader turns; at the same time the protons lose energy and they finally turn around quite sharply. Conservation of energy and momentum requires that the protons and electrons exchange energies momentarily (i.e., $u_y(\max) = u_- \approx (m_p/m_e)^{1/2}u_o)$ but that they regain their original energies when they leave the sheath. To order $(m_e/m_p)^{1/2}$ the sheath thickness is $\delta' = c/\omega_p$ (electron) but since $2N_o m_p u_o^2 \simeq B^2/8\pi$, δ' is on the order of $u_o/\sqrt{\omega_c^e\omega_c^p}$. (Dungey, 1958).

The self-consistency and uniqueness of the minimum sheath for this hydrogen plasma-field interface might appear to be established to the same extent as in the equal mass case, if an external source of trapped particles is not present. For instance, although electric fields exist and transient energy transfer does occur (see Fig. I), this simple theory still predicts overall conservative and specular reflection. Thus, if $f(\underset{\sim}{v},\underset{\sim}{x}) = f^*(P_y, \mathcal{E})$ represents only the incident particles on collisionless trajectories, it would again seem that the only current which can form is the one needed to shield out the main field.

In fact, Piddington (1960) showed that this picture must be modified because the sheath has an inherent plasma instability. Figure I shows that in the hydrogen plasma sheath the electrons and protons have a relative drift velocity of magnitude $V = \left[(m_e/m_p)^{1/2} + (m_p/m_e)^{1/2} \right] u_o$ in the direction of $\underset{\sim}{E}_{cs} \times \underset{\sim}{B}_o$, where $\underset{\sim}{E}_{cs}$ is the charge separation electric field. This drift can excite the longitudinal modes ($\underset{\sim}{k} \times \underset{\sim}{E}' = 0$, $\underset{\sim}{k} \times \underset{\sim}{V} << |k| \cdot |V|$) of oscillation of the plasma. In order to investigate this we assume that the magnetic field is effective in setting up the finite drift velocity, but that the $B_o = 0$ dispersion relation is adequate to describe the longitudinal modes.

This dispersion relation, for a wave of frequency ω, wave number $\underset{\sim}{k}$, is

$$1 = \frac{\left(\omega_p^e\right)^2}{k^2} \int \frac{d^3 v}{(\underset{\sim}{k} \cdot \underset{\sim}{v} - \omega)} \left[\underset{\sim}{k} \cdot \underset{\sim}{\nabla}_v f_e + \left(\frac{m_e}{m_p}\right) k \cdot \nabla_v f_p \right] \qquad (1)$$

with $\mathrm{Im}\,\omega > 0$, plus the analytic continuation to the lower half ω-plane. In the absence of drift, there are two principal branches, the electron plasma oscillations, and the ion "sound" waves. For instance, if the distributions are

$$\bar{f}_{e,p} = \int (d^2v)_\perp \; f_{e,p}(\underset{\sim}{v}) \simeq \frac{(a,A)}{\pi\left[(a,A)^2 + v_\parallel^2\right]} \quad , \qquad (2)$$

with $\underset{\sim}{k} \cdot \underset{\sim}{v} = k\, v_\parallel$, etc., then the long wavelength ($ka << \omega_p^e$) solutions (Fried and Gould, 1961) to Eq. (1) are

$$\omega_1 \simeq \omega_p^e \left[1 + m_e/2m_p\right] - i\, ka \qquad (3)$$

and

$$\omega_2 \simeq ka\sqrt{\frac{m_e}{m_p}} - i\, kA \quad . \qquad (4)$$

Although these distributions are quite unrealistic, the main properties of the modes are represented in Eqs. (3) and (4). The electron plasma oscillations (ω_1) are undamped in the limit of zero electron temperature, while the ion waves (ω_2) are very heavily damped, and hence do not propagate unless $\Theta = T_e/T_p = m_e a^2/m_p A^2 >> 1$.

Let us now consider the effect of drift. In the electron rest frame f_e is unchanged, but

$$f_p = \int (d^2v)_\perp f_p \longrightarrow \frac{A}{\pi\left[(v_\parallel - V)^2 + A^2\right]} \quad , \qquad (5)$$

and Eq. (1) becomes

$$\frac{1}{\left|\omega_p^e\right|^2} = \frac{1}{(\omega - iak)^2} + \frac{m_e}{m_p}\,\frac{1}{(\omega - kV - ikA)^2} \quad . \qquad (6)$$

If $m_e a^2 = m_p A^2$, an exact undamped (real) solution has $\omega(1 + m_e/m_p) = kVA$, and $(\omega^2 + k^2 a^2)^2 = 2\omega^2(\omega^2 - k^2 a^2)$. These equations are compatible for real ω only if $V \geq V_c = a(1 + \sqrt{m_e/m_p})$, and V_c thus defines the limit of neutral stability. For $V < V_c$, the waves are damped, and if $V > V_c$, growing waves with $0 < k < \omega_p/2a$ are generated [for an equal temperature hydrogen plasma with maxwellian distributions, $V_c = 0.925\, a(1 + \alpha^{1/2})$, $\alpha = m_e/m_p$.] This is just a modification of the ordinary two-stream instability for excitation of electron plasma oscillations ($\omega \simeq \omega_p^e$), and Piddington first observed that this mechanism is relevant in the magnetopause. The two-stream instability occurs if the charge separation fields induce a drift with $V > a$; however, $V \approx \alpha^{-1/2} u_o$, and thus for equal electron and proton temperatures the criterion becomes $m_p u_o^2 > kT_e = kT_p$. This condition always seems to be satisfied in the solar wind [$u_o \geq 300$ km/sec, $T \lesssim 10^6\,^\circ K$ so that $m_p u_o^2 \geq 10\ kT$].

Once the onset of instability in the sheath is established, it is important to analyze its temporal development. However, since the electric fields grow as $\exp(\omega' t)$, one is immediately led into the nonlinear regime where the distribution functions become distorted, and the applicability of the dispersion relation [Eq. (1)] is questionable. It is conventional to resolve this problem by invoking conservation of total energy (streaming, thermal and electric). The energy is presumably transferred by scattering of particles from the fluctuating space charge fields [$E'(t)$] associated with the wave instabilities. The scattering should reduce the drift energy, and increase primarily the electron thermal energy; it is assumed throughout that the electrons provide the main heat sink and that the ion temperature is essentially unchanged. Thus, V decreases with time, a increases, and $V^2(t) + a^2(t)$ is approximately constant.

These concepts, together with the original instability criterion ($V > 0.925\ a$), were considered by Piddington, and he concluded that the instability will quench itself with $V_{final} \simeq a_{final}$; this is a familiar result associated with the self-quenching of an electron-electron or ion-ion double stream instability, and it leads to a sheath thickness on the order of an ion gyroradius. However, subsequent analyses have revealed that this criterion does not adequately describe the growing modes of an electron-proton plasma with finite drift and unequal temperatures. In fact, the critical drift speed drops sharply with increasing θ as the ion sound wave modes become unstable.

At this point, it is appropriate to turn to more realistic Maxwell distributions since the dependence of V_c on θ is very sensitive to the shape of $f_p(v)$ in the tail region, $v^2 >> A^2$. In this case, Jackson (1960), and Fried and Gould (1961), have computed $V_c(\theta)$ and a curve based on their results is shown in Fig. II. For fixed A, $V_c(\theta)/A = .925\ (1 + \alpha^{1/2})/\alpha^{1/2} \simeq 40$ at $\theta = 1$ but $V_c(\theta)/A$ is reduced to 4 for $\theta >> 20$. Thus, if T_p remains fixed and T_e increases with time, then as V_c drops the oscillations are not quenched at $V_c(t) \simeq a\ (t) >> a(0)$. Instead the drift threshold falls to $V_c(t) \simeq 4A(t) = 4A(0)$.

In order to apply this, we again assume conservation of electron kinetic energy, and we also assume that the final state is determined either by intersection of $V(\theta)$ with the stable boundary, or by a form of equipartition of solar wind energy (i.e., constant T_p), with the lower of the two electron temperatures prevailing. Since $\alpha a^2(\theta) = \theta A^2$, and $V(1) \simeq u_o/\alpha^{1/2}$, the energy equation gives

$$\left[V(\theta)/A\right]^2 \simeq \frac{1}{\alpha}\left\{(u_o/A)^2 - \theta + 1\right\} \quad , \qquad (7)$$

while energy"equipartion" yields $(kT_e)_{max} = \frac{1}{2}(m_p u_o^2/2)$. Let us now consider a wind with u_o = 600 km/sec, A = 40 km/sec, and a(initial) = 1720 km/sec $\left[T_p = T_e(\text{initial}) \simeq 2 \times 10^{5\,o}\,K\right]$. If the electron heating is limited by the onset of stability, then $V(\theta_{final}) \simeq 4A$ (see Fig. II), and Eq. (7) gives $\theta_{final} \simeq 226$, $T_e(\text{final}) \simeq 5 \times 10^{7\,o}$ K. However, the "equipartition" relation leads to $T_e(\text{max}) \simeq 1.1 \times 10^{7\,o}$ K. Thus, the final state should remain unstable with respect to ion wave generation, and the sheath should contain finite currents. It must be noted that these numerical results, and the simplified energy relations are highly unrealistic. Nevertheless, the above arguments strongly suggest that the C-F sheath contains plasma instabilities of the ion-wave variety which are not quenched by nonlinear effects.

3. MAGNOTOPAUSE BROADENING MECHANISMS

If it is indeed true that the magnetopause is unstable with respect to generation of electric field oscillations associated with the ion wave mode, then several new phenomena become significant. The fluctuating electric fields allow the incident plasma to diffuse through the idealized C-F geomagnetic boundary toward the cavity interior. As the sheath broadens and becomes disordered, the reflection becomes diffuse instead of specular, and ultimately the particles undergo deflection, rather than reflection. This development has several important consequences. First, since the scattered or diffusely deflected particles are not returned to the solar wind with constant energy and canonical momentum, the conventional proof that the self-consistent magnetic profile is "thin" breaks down completely. The deflected particles have an effect analogous to the presence of a trapped ring current. That is, since the scattered plasma particles are present in the magnetopause for a finite time, the distribution function contains two independent contributions: incident and "quasi-trapped" scattered plasma. As we shall see, the sheath also tends to diffuse back toward the wind, and since the field is determined by the current distribution, the self-consistent magnetic profile will tend to have a broad and relatively smooth transition from $|\underset{\sim}{B}| = B_G$ to $|\underset{\sim}{B}| = B_I$.

Finally, the fluctuations allow rapid diffusion of thermal plasma outward from the geomagnetic cavity. This contributes to the "quasi- trapped" distribution and enhances the broadening of the magnetopause. Since the C-F sheath is then spread over a large transition region, the instability, the field fluctuations and the hot electrons should be observed over the entire region.

It is very difficult to translate these rather general speculations into detailed predictions of local magnetic field profiles, electric field energy distributions and electron fluxes. Part of this difficulty arises from the intrinsic nonlinearity of the ion-wave instability. Smith and Dawson (1963) recently examined some nonlinear effects and growth rates for an artificial one-dimensional plasma model with $\alpha = m_-/m_+ = 1/25$ and T_- fixed at $10T_+$; in this case the distortions associated with the ion-wave instability produce a final distribution which appears to be stable but "noisy". However, these results cannot be extrapolated to the magnetopause case with $\alpha^{-1} = 1840$ and variable θ. Moreover, the presence of a stationary thermal plasma within the cavity seriously complicates any attempt to construct the self-consistent magnetic profile since all particles respond to the total local field and can contribute to the field.

In fact, several specific features of the earth's magnetopause must be related to nonuniformities in the solar wind and unperturbed geomagnetic field, to the earth's rotation, and to the orientation of the geomagnetic field. In this connection we refer primarily to the abrupt near reversal of direction of the total field frequently seen in the sub-solar region on Explorer 12 (Cahill and Amazeen, 1963). An apparent neutral point at $(8 - 9)R_E$ could be associated with the electric field induced by the rotation of the earth and magnetosphere below the solar wind. The reversal in field direction might simply be related to the fact that the external current system is free to rotate and that the state of the lowest energy has anti-parallel dipole moments. It is also conceivable that the configuration is a direct consequence of the nearly 90° relative orientation of $\underset{\sim}{B}_G$ and $\underset{\sim}{B}_I$. On the other hand, these changes in direction may be completely unrelated to the predictions of any steady-state interaction theory.

At this time we are not in a position to resolve this problem which clearly depends on the three-dimensional nature of the true dynamic solar wind-interplanetary field-magnetosphere interface. Nevertheless, it is possible to supplement the above qualitative conjectures with some quantitative data on diffusion rates, densities, etc. It is well known that a collisionless plasma can rapidly diffuse across a magnetic field (Bohm, 1949). An empirical expression for this "fast" diffusion speed is

$$v_{\perp} = \frac{+c}{16\ NeB}\ \nabla P_e = \frac{4 \times 10^{18} \nabla P}{NeB} \left(\frac{cm}{sec}\right) \qquad (8)$$

where $P_e = N_e kT_e$; Bohm, and Spitzer (1956), speculated that some kind of oscillation with randomly varying electrostatic fields may yield Eq. (8). Spitzer (1960) later attributed this diffusion to the ion-wave instability which, as has been shown, occurs when the ions are relatively cold and a small current is present; an approximate semi-empirical diffusion equation based on stellarator data is

$$\langle (\Delta x)^2 \rangle = 0.21\ (ckT_{eff}/eB)t \quad , \qquad (9)$$

with T_{eff} undetermined but bounded by the actual electron and proton temperatures. This fast diffusion produces severe magnetopause modifications.

For instance, if $u_o = 500$ km/sec, $N = 5\ cm^{-3}$ and $T_p = T_e$(initial) $= 2 \times 10^5$ ^{o}K, then the sub-solar C-F sheath, which requires $(B)^2/8\pi = B_G^2/2\pi \simeq 2Nm_p u_o^2$, yields $B_G = 5 \times 10^{-4}$ gauss. The cavity boundary is located at $r = 8.6\ R_E$ and the sheath thickness is $\delta = \sqrt{2}\ u_o/(\omega_c^+\omega_c^-)^{1/2} \simeq 1 - 2$ km. In the case of diffuse reflection, the same wind is stopped at $r = 9.6\ R_E$ ($B \simeq 7.2 \times 10^{-4}$ gauss) and one might estimate δ(diffuse) $\simeq u_o/\omega_c^+ \simeq 70$ km. However, Eq. (9) gives

$$\langle [\Delta x\ (\mathrm{km})]^2 \rangle \simeq 50\ (T_{eff}/T_p)t(\mathrm{sec}) \qquad (10)$$

so that if $T_{eff} = T_p$, then at 1000 sec after formation $|\Delta x| \simeq 220$ km, and if T_{eff} is $\theta_{max} T_p = T_e(\max) \simeq 10^7$ ^{o}K, then $|\Delta x| \simeq 2000$ km, by this time. Thus, a stationary thin C-F sheath cannot possibly be maintained in the presence of fast diffusion (i.e., finite currents and hot electrons).

Of course, as the current sheath broadens, the magnetic profile does the same, since the sheath particles interact electromagnetically with the incident wind and with the total local field. Grad (1961) has emphasized that the self-consistent equilibrium solutions for a plasma-field interface are extremely sensitive to the presence of trapped (or in our case, quasi-trapped) orbits, and that an arbitrary magnetic field profile can be reproduced in a self-consistent calculation by placing trapped particles on magnetic field lines in the appropriate manner. For instance, in the two-dimensional model for which $P_y = mu_y + eA_y(x)$ is always a constant of motion, Grad shows how to construct a total distribution function $f(P_y) \longrightarrow f[A_y(x)]$ which is consistent with any given $B(x)$. In essence, the trapped particles cluster into groups (in configuration and velocity space) which produce currents and modify the fields; the clustering depends on the effective orbits of the quasi-trapped particles (including source and depletion rates), as well as on the interaction with free particles and with the self-consistent field, but the presence of quasi-trapped particles generally precludes the formation of a minimum profile with a smooth monotonic transition between incident plasma and applied field.

An additional source of quasi-trapped particles in the transition region is provided by outward diffusion of the magnetosphere plasma. Whistler analysis (Liemohn and Scarf, 1963) suggests $N_e(r) \simeq 1.4 \times 10^4 (R_e/r)^3$, $2.5 < r/R_e < 5$ and it is tempting to extrapolate this distribution to the outer magnetosphere to yield $N_e(8R_e) \simeq 27\ \mathrm{cm}^{-3}$, for instance. Actually, this procedure has significant uncertainty since some evidence of an abrupt break in the $N_e(r)$ curve has been presented (Carpenter, 1963a). However, this break may very well be associated with magnetic storms, instead of being a permanent feature of the magnetosphere (Liemohn and Scarf, 1963), and whistlers propagating to altitudes as high as $8R_e$ have recently been detected (Carpenter, 1963b). For these reasons, we assume a significant thermal plasma concentration (say $N_e \simeq 10 - 30\ \mathrm{cm}^{-3}$) just below the magnetopause.

Equation (8) may be used to demonstrate that this plasma cannot be contained within the cavity. If the upper magnetosphere temperature is near $10^{5}\,{}^{\circ}\mathrm{K}$, then the containment time of a thin sheath is on the order of

$$t_c = \int_x^{x+\delta} \frac{dx}{v} \simeq 1.8 \times 10^{-6} \langle B \rangle \delta^2 , \qquad (11)$$

where $\langle B \rangle$ is in gauss and δ in centimeters. For the specular C-F sheath ($\langle B \rangle = B_G(r_o) \simeq 5 \times 10^{-4}$, $\delta \simeq 2 \times 10^5$) Eq. (11) yields $t_c \simeq 40$ sec and for the "diffuse" C-F sheath ($\langle B \rangle \simeq 3.6 \times 10^{-4}$, $\delta \simeq 7 \times 10^6$) it gives $t_c \simeq 3.2 \times 10^4$ sec. Thus, there is no steady-state containment of thermal plasma, and the magnetopause is populated with an exospheric distribution of low energy particles.

The steady-state configuration should be related to a modified local pressure balance equation,

$$\sum (NkT + Nmu^2) + \frac{B^2}{8\pi} - \frac{E_{cs}^2}{8\pi c^2} + \frac{E_w^2}{8\pi c^2} = \text{const.} \quad , \tag{12}$$

where the sum includes incident, scattered and exospheric protons and electrons, E_{cs} is the local charge separation electric field (parallel to u_o) in e.m.u. and E_w is the ion-wave electric field (parallel to V); B is the total self-consistent magnetic field. Equation (8) suggests that this configuration will adjust itself so that the mean value of $\nabla(NkT_e)$ is as small as possible, and Eq. (12) then implies that the average of ∇B tends to a minimum. If the unstable region starts at some specific inner radius, r_o, then Eqs. (8), (12) predict a smooth gradual transition for $|B_{av}|$ from $(1\text{-}2)B_G(r_o)$ at $r = r_o$ to B_I as $r \longrightarrow \infty$. In other words, the theory based on current instabilities and fast diffusion does not yield a finite outer magnetopause boundary; instead, a gradual transition to interplanetary fields, densities, etc. is predicted.

Of course, these considerations apply only to the mean fields and densities. If fast diffusion is to operate over a large range, then the instability must be present throughout. Thus, it is anticipated that large deviations from the mean will be found over the entire transition region, with fluctuation lengths greater than or equal to the local ion Debye wavelength, $\lambda_o = (kT_p/4\pi Ne^2)^{1/2} \simeq$ 10 - 20 m. (It is noteworthy that λ_o is very much smaller than any gyroradii in a field of $10^{-4} - 10^{-5}$ gauss. An even more significant point is that λ_o is negligible compared to any of the possible initial sheath thicknesses; thus the ion-wave oscillations can occur freely.)

The final question concerns the location of the inner boundary of the disordered diffuse sheath. This limit should be related to the efficiency of the electron heating process, since the current instability will persist only if E_{cs} is effective in increasing θ so that $V(\theta) > V_c(\theta)$ for $\theta = \theta_{final}$ (see Fig. II). Clearly, if other local heat sinks are present, then V can be lowered without increasing T_e to the extent necessary to maintain the instability, and Eq. (7) becomes invalid. We suggest that the whistler medium itself provides the heat sink which quenches the instability, and we assert that in the sub-solar region this transition should occur when $P(\text{whistler}) = NkT > P(\text{wind}) \simeq \frac{1}{2} Nm_p u_o^2$ (ion heating could also terminate the instability). If the extrapolated density distribution is correct and if the upper whistler medium temperature is on the order of 2×10^{5} °K, then during quiet times r_o might be $(8 - 10)R_E$. However, the quenching altitude is obviously quite variable, it is sensitive to solar activity and changes in solar wind flux, and it depends on latitude and longitude. Our main point is that an abrupt transition to a stable configuration is to be expected at all times. Figure III shows the anticipated electron density and flux distributions and the envelope of the expected magnetic field magnitude.

At first glance, the prediction of Fig. III would seem to be in good agreement with observations made on Explorer 12. For instance, on the September 13 inbound pass an abrupt transition was found near the sub-solar point at $r_o = 8.2\ R_E$ (Cahill and Amazeen, 1963). Below this point the compressed geomagnetic field was relatively stable and the high energy spectrometers detected large fluxes of conventional trapped radiation (Freeman, Van Allen and Cahill, 1963). Beyond r_o the high energy counters detected only the normal cosmic-ray background, and the magnetic field was characterized by rapid spatial and/or temporal variations. Furthermore the response of the CdSTE detector indicated the presence of an isotropic flux of kilovolt electrons beyond r_o, which resulted, somehow, from the interaction of the solar wind with the geomagnetic field. Indeed, the CdSTE flux and the absolute magnitude of $|B|$ had large fluctuations superimposed on a broad, monotonic decrease as in Fig. III.

However, the perplexing Explorer 12 observations of changes in the <u>direction</u> of B are not explained by our theory, as mentioned above. Although direct measurements of electron density were not performed on Explorer 12, the high electron temperatures observed already suggest quite strongly that the sheath should have a current instability, but it must be noted that since the ion temperature in this region is not known, we have no direct confirmation of our prediction $T_e/T_i >> 1$. On the other hand, the Pioneer 5 magnetometer measured disturbances extending out to 30 R_E in the sunlit hemisphere which were apparently related to the presence of the geomagnetic field (P. Coleman, private communication); this extensive observation of earth-oriented magnetic activity again tends to support the current instability - fast diffusion theory of sheath broadening.

4. DISCUSSION

The theoretical mechanism proposed above is based on experimentally observed laboratory plasma-field interactions but its applicability to the nonuniform, dynamic magnetopause is highly speculative and qualitative. However, in view of the existence of finite density, field and temperature gradients it is certain that some form of "universal" instability must be triggered (Krall and Rosenbluth, 1963) so that even if the detailed description of the approach to quasi-equilibrium differs from the one presented in Section 3, the final state should have the characteristics shown in Fig. III.

Of course, other sheath-broadening mechanisms which do not depend on instabilities have been proposed from time to time, and one must consider the extent to which they remain relevant. The simplest model, emphasized by Alfvén (Karlson, 1963) treats the solar wind as a highly conducting dielectric. As the plasma intrudes on the field, currents begin to flow within the sheath, but it is asserted that this transport of charge (which implies space charge formation on some surfaces) will not continue indefinitely. Ultimately a Hall electric field develops anti-parallel to the current, which then tends to zero. After this balance is achieved, the subsequent streams pass into the magnetosphere with an $\underset{\sim}{E} \times \underset{\sim}{B}$ drift, and they are then acted upon by forces associated with field gradients. In fact, our model is related to this one, but we regard the initial current as being sufficiently large to trigger the ion-wave instability. In this case, the <u>fluctuating</u> Hall fields allow diffusion, not free penetration, and no steady space charge is maintained.

In the other popular models the interplanetary field, B_I, plays an essential role by coupling the solar wind into a fluid on a scale small compared to the diameter of the magnetosphere, $d \simeq 1.3 \times 10^5$ km. Thus continuum rather than free flow theories are used to describe the interaction and the value of the characteristic Alfvén speed, $V_A = B/(4\pi Nm)^{1/2} \simeq 100$ km/sec for $B_I \simeq 10^{-4}$ gauss, $N \simeq 5$ cm^{-3}, becomes significant. Two cases can be distinguished. If the speed with which the earth moves across the interplanetary field and plasma is between V_A and $2\ V_A$, then an "Adlam-Allen pulse" type of sheath (Adlam and Allen, 1958) is formed. This initially resembles the ordered monotonic C-F sheath but it is somewhat broader, say $\delta \simeq (1 - 2)\ \delta_{CF}$ (Colgate, 1962). However, if the Mach number is greater than two, then the "supersonic" motion may generate a standing or detached shock wave whose stagnation point is located at (12 - 14) R_E (Axford, 1962; Kellogg, 1962). Behind this collisionless shock one would expect disordered streaming with most of the energy appearing in the form of random thermal motion. (Presumably instabilities again convert streaming energy to thermal energy.) Moreover, two-dimensional model magnetohydrodynamic calculations

(Auer, Hurwitz and Kilb, 1962) indicate that beyond the shock the magnetic field does not make a smooth transition from B_I to an intermediate value, but rather that it oscillates about some average.

We feel that this theory, which is primarily based on hydrodynamic analogies, may not be applicable to the magnetopause. The collisionless shock has never been unambiguously observed in the laboratory, and some experimental studies of plasma chock waves (Kantrovitz, Patrick and Petschek, 1960) indicate that the disordered interaction region extends only a distance of several ion-gyroradii (not 20,000 - 30,000 km) behind the shock. On Pioneer 5 (P. Coleman, private communication) no firm evidence for the existence of a shock front with quiet conditions upstream was obtained. In fact, there is some doubt about the way to apply this theory to the solar wind-magnetosphere interface. It is customarily assumed that the solar wind speed should be used in computing the Mach number, but the theoretical shock models require that the particle orbits not be influenced by any electric fields. In Parker's model of the interplanetary field (1958b), $E = 0$ only in the frame rotating with the sun, and in this frame u_o and B_I are parallel and inclined at the hose angle (approximately 45^o near the earth). If this frame is indeed appropriate then the component of the relative earth-plasma speed perpendicular to B_I is on the order of the earth's orbital speed (about 230 - 270 km/sec); the Mach number is quite low, and M frequently falls below two so that the initially smooth "Adlam-Allen sheath" results. Instabilities similar to those described above are then expected for the Adlam-Allen interaction.

Finally, we observe that near the earth where the solar wind has $\ell(dT/dr) \gtrsim T$ and $|T_\perp - T_\parallel| \neq 0$ (Scarf, 1963) sufficient disorder may be present to make any continuum model questionable over distances on the order of 10^5km (the size of the magnetosphere) and times on the order of 10^3 sec (since $V_A \simeq 100$ km/sec, this is the time needed to set up the shock with a standoff distance of $4R_E$); Eqs. (10) and (11) also indicate that the current instability and the fast diffusion will induce considerable broadening and distortion before such a shock can form.

Thus we feel that the broad disordered transition between the solar wind and the magnetosphere is associated with instabilities in the C-F sheath. It remains to construct a more quantitative model, to extend the theory away from the sub-solar region and to explain the $\underset{\sim}{B}/|B|$ variation in the transition region. Furthermore, certain nonlinear wave-wave interactions (such as ion-wave resonance with the electron gyrofrequency) are possible (Stix, 1963) and these effects must be examined.

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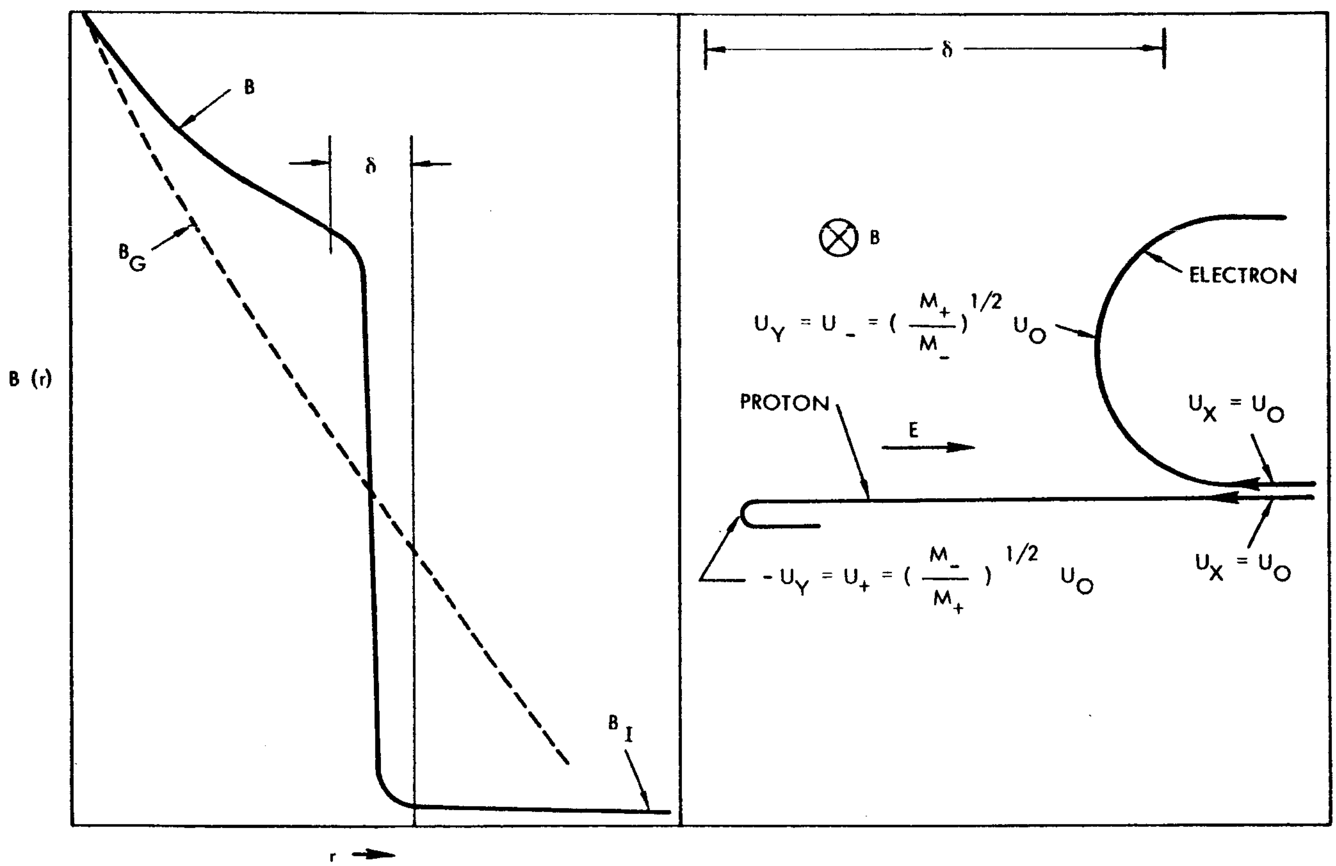
Figure Captions

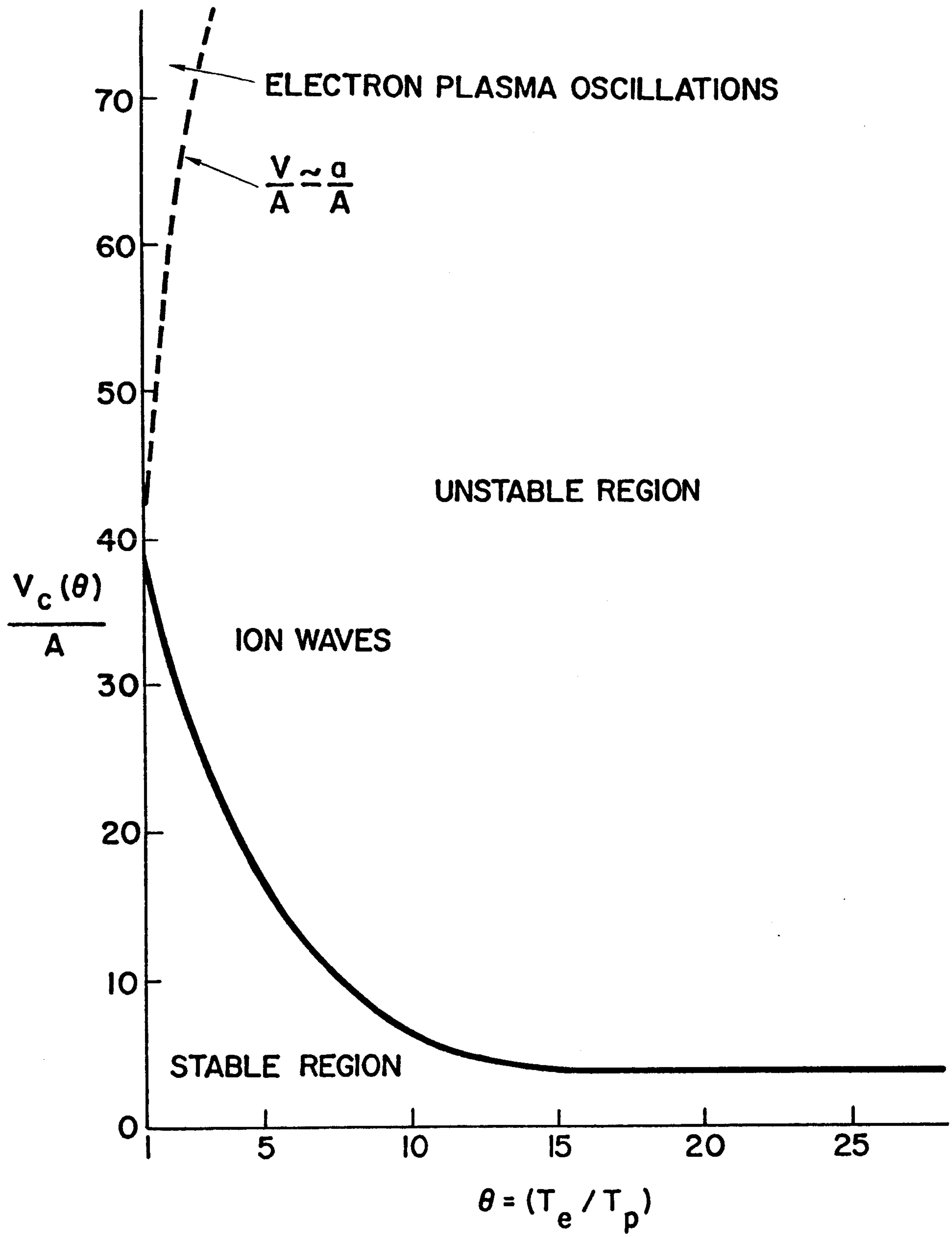
Fig. I. The idealized thin C-F sheath which separates the magnetosphere ($|B| = (1\text{-}2)B_G$) from the solar wind and the interplanetary field ($|B| = B_I$). The charge separation electric field, E, pulls the electron into a broad orbit and slows the proton until the latter suddenly turns.

Fig. II. The critical drift velocity, V_c, as a function of the electron-proton temperature ratio, θ. Here $m_e a^2 = kT_e$, $m_p A^2 = kT_p$, and for a given θ, any velocity above the heavy line represents an unstable configuration. The conventional two-stream instability occurs to the left of the dotted line ($V \approx a$) and the ion waves are unstable to the right of this line. These curves are based on the dispersion equation for a collisionless, maxwellian hydrogen plasma.

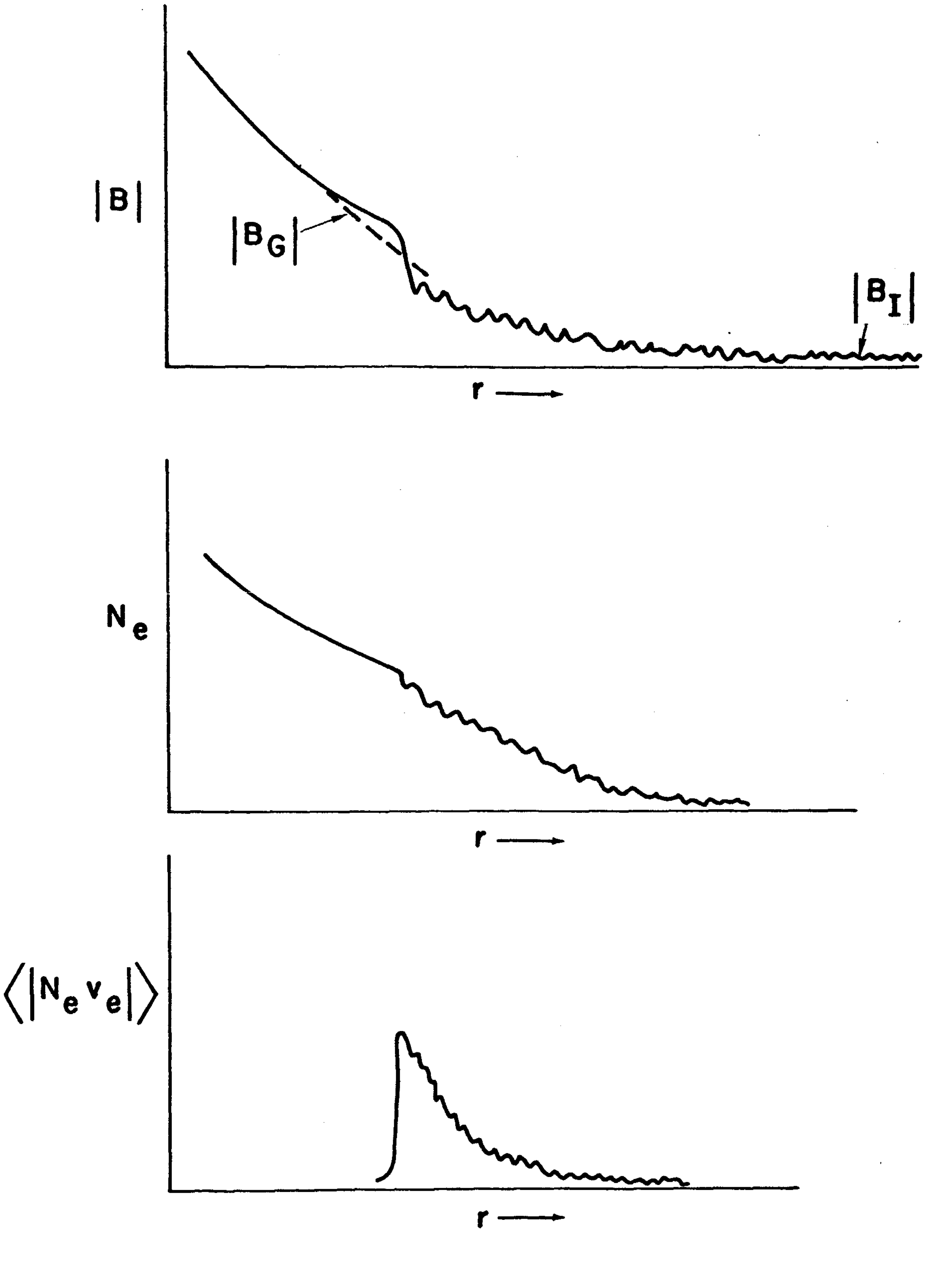
Fig. III. The anticipated profiles of the absolute value of the magnetic field, the electron density and the electron flux, along the noon meridian. Here B_G and B_I represent the undisturbed geomagnetic and interplanetary field magnitudes, N_e represents the density extrapolated from whistler measurements, and $\langle | N_e v_e | \rangle$ includes drift and thermal motion.



B
δ
B G
B (r)
B I
r
δ
B
ELECTRON
U Y = U - = (M+ / M-) 1/2 U O
PROTON
E
U X = U O
- U Y = U + = (M- / M+) 1/2 U O
U X = U O



ELECTRON PLASMA OSCILLATIONS
$\frac{V}{A} \simeq \frac{a}{A}$
UNSTABLE REGION
ION WAVES
STABLE REGION
$\frac{V_c(\theta)}{A}$
70
60
50
40
30
20
10
0
1
5
10
15
20
25
$\theta = (T_e / T_p)$



|B|
|B_G|
|B_I|
r →
N_e
r →
⟨|N_e v_e|⟩
r →